$$x_n \geq \frac{3}{2} \Rightarrow \frac{1}{x_n} \leq \frac{2}{3} \Rightarrow \frac{1}{\varphi.x_n} \leq \frac{2}{3} \times \frac{1}{\varphi} \Rightarrow \frac{1}{\varphi.x_n} \leq \frac{\sqrt{5}-1}{3} \Rightarrow \frac{1}{\varphi.x_n} \leq \frac{3(\sqrt{5}-1)}{9} \Rightarrow \frac{1}{\varphi.x_n} \leq \frac{4}{9}$$

d'où $|x_{n+1} - \varphi| \leq \frac{4}{9}|x_n - \varphi|$

d) $|x_1 - \varphi| \leq \frac{4}{9}|x_n - \varphi|$

$|x_2 - \varphi| \leq \frac{4}{9}|x_1 - \varphi|$

$|x_3 - \varphi| \leq \frac{4}{9}|x_2 - \varphi|$

.

.

$|x_n - \varphi| \leq \frac{4}{9}|x_{n-1} - \varphi|$

multiplions membre à membre et simplifions on a :

$$|x_n - \varphi| \leq (\frac{4}{9})^n |2 - \varphi|$$

$$\lim_{n\to+\infty} (\frac{4}{9})^n = 0 \Rightarrow \lim_{n\to+\infty} (x_n - \varphi) = 0 \Rightarrow \lim_{n\to+\infty} x_n = \varphi$$

EXERCICE№14

$U_n = \sum_{k=1}^{n} \frac{1}{k^2}$; $n \in \mathbb{N}^*$

1) a) $U_1 = \sum_{k=1}^{1} \frac{1}{k^2} = 1$; $U_2 = \sum_{k=1}^{2} \frac{1}{k^2} = 1 + \frac{1}{2^2} = \frac{5}{4}$; $U_3 = U_2 + \frac{1}{3^2} = \frac{5}{4} + \frac{1}{9} = \frac{49}{36}$

b) $U_{n+1} - U_n = \frac{1}{(n+1)^2} \geq 0$ d'où (U_n) est croissante

2) a) $k \geq 2$

$$(\frac{1}{k-1} - \frac{1}{k}) - \frac{1}{k^2} = \frac{1}{k(k-1)} - \frac{1}{k^2} = \frac{1}{k^2(k-1)} \geq 0 \quad \text{d'où} \quad \frac{1}{k^2} \leq \frac{1}{k-1} - \frac{1}{k}$$

b) $n \geq 2$ $k \geq 2$, on a : $\frac{1}{k^2} \leq \frac{1}{k-1} - \frac{1}{k}$